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HEAT EMISSION ACCOMPANYING THAWING OF A VERTICAL ICE SURFACE

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UDC 551.468.1:536.24

A dependence is proposed for determining the heat-transfer coefficient at the boundary of an ice massif and a water film running down the ice in the presence of thawing.

Heat transfer from air to an ice massif during the summer occurs through the liquid film which is formed and runs down the surface of the ice. This films plays the role of a thermal insulator for the ice massif. It runs down under the action of gravity, and its flow has a wave character [1, 2]. The thickness of the film varies over the height, so that the coefficient of heat transfer must be regarded as variable, which causes nonuniform thawing of the ice massif: thawing will be greater at the top than at the bottom.

Thawing of vertical surfaces was studied previously in [3, 4], but the factor responsible for this process was assumed to be film condensation. This enabled assuming that the boundary temperature at the outer surface of the thawing medium is equal to the condensation temperature. The solutions are presented in a form such that a computer is required to obtain numerical results; in addition, the deviation of the results from the exact solution under some conditions reaches 20% [4].

Figure 1 shows the diagram for the calculation of the magnitude of the thawing of a vertical wall of an ice massif.

The heat flow from the air to the liquid film formed is determined by the expression

$$q_{\text{air}} = \alpha_{\text{air}} (T_{\text{a}} - T_s). \tag{1}$$

This heat is transferred through the liquid film to the ice massif, with the exception of a small fraction expended on increasing the heat content of the liquid volume.

The heat flow through the liquid film in the section x is equal to

$$q = \alpha_x (T_s - T_{\text{m}}). \tag{2}$$

This quantity can be expressed differently for a laminar fluid motion:

$$q = \frac{\lambda}{\delta_x} (T_s - T_{\text{m}}). \tag{3}$$

V. P. Chkalov Gor'kov Civil Engineering Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 6, pp. 1034-1038, December, 1985. Original article submitted November 16, 1984.

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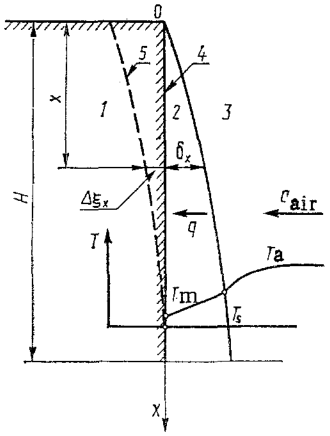

Fig. 1

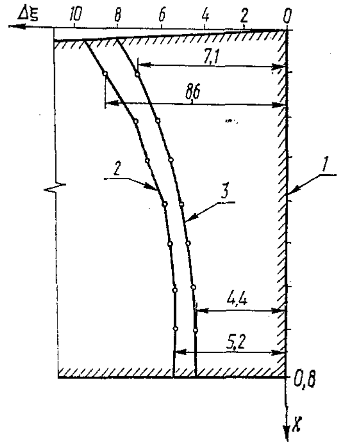

Fig. 2

Fig. 1. Flow of a water film along the surface of a thawing ice massif: 1) ice; 2) water film running off; 3) air; 4) ice boundary before the onset of thawing; 5) same during the thawing process.

Fig. 2. Comparison of the experimental results and the calculation of thawing of an ice plate: 1) ice surface prior to the beginning of the experiments; 2) 2.5 h after the end of the experiment; 3) results of the calculation. x, m; $\Delta\xi$, mm.

The air temperature T_a and the temperature of the outer surface of the liquid film formed T_s are assumed to be constants, and the latter is unknown.

From a comparison of expressions (2) and (3) we can write

$$\alpha_x = \lambda/\delta_x. \tag{4}$$

To determine the heat-transfer coefficient it is necessary to find the thickness of the liquid film formed.

Nusselt studied the laminar flow of a liquid along a vertical wall during condensation of vapor [5]. If Nusselt's arguments are applied to the flow of a water film down the vertical surface of a thawing ice massif, then the following expression is obtained for determining the thickness of the film:

$$\delta_x = \sqrt[4]{\frac{4\mu\lambda x (T_s - T_m)}{\gamma_i L \gamma}}. \tag{5}$$

As the studies showed, the flow of the liquid film along the wall is a wavey flow, and its average thickness decreases by 13%, while the effectiveness of heat transfer increases on the average by 21% [1]. Taking this circumstance into account and substituting (5) into (4), we can write the expression for determining the local value of the heat-transfer coefficient at the boundary of the liquid film with the vertical ice massif as follows:

$$\alpha_x = 1.21 \sqrt[4]{\frac{\lambda^3 \gamma_i L \gamma}{4\mu x (T_s - T_m)}}. \tag{6}$$

The average value of the heat-transfer coefficient along the entire height of the surface H is equal to

$$\alpha_{av} = \frac{1}{H}\int_0^H \alpha_x dx = \frac{4}{3}\, 1.21 \sqrt[4]{\frac{\lambda^3 \gamma_i L \gamma}{4\mu H (T_s - T_m)}}. \tag{7}$$

To check these calculations, we performed an experiment on thawing of a vertical ice plate.

A plate of ice formed from distilled water was frozen in a freezing chember in a boat in layers up to 2 mm at a temperature of 1-2°C. Its final size was 0.134 × 0.8 × 0.045 m. Prior to testing the plate was held for a day at a temperature of 0°C. Then it was carried into an enclosure at room temperature and placed in a strictly vertical position. The thawing process began immediately. After definite time intervals, with the help of a water gauge, the magnitude of the thawing was measured along the height of the plate to within 0.1 mm (Fig. 2). At the same time, the air temperature was measured with the help of a laboratory mercury thermometer with scale divisions of 0.1°C. The thermometer was placed at the center of the plate along the vertical at some distance from the surface of the plate, where the cooling action of the ice did not affect the air medium.

During the experiment only the free motion of air along the surface of the water film running down the ice at a temperature T_a occurred. For this reason, radiant and convective heat transfer were observed between the air medium and the liquid film running down the ice.

The ice plate was slightly sloping at the top so that the water from the top boundary would not flow onto the face of the plate during thawing.

The experimental measurements of the thawing are compared with the results of the calculations. The physical constants of air, ice, and water are taken from handbooks [6]. Because of the fact that it was technically impossible to fix the temperature T_s, it was determined by calculation. For this, we used the relation obtained from the conditions (1) and (2):

$$\alpha_{\mathrm{air}}\,(T_a - T_s) = 1.21\,\frac{4}{3}\sqrt[4]{\frac{\lambda^3\gamma_i L\gamma}{4\mu H\,(T_s - T_m)}}\,(T_s - T_m). \tag{8}$$

The solution was sought by trial and error. For this the temperature T_s was given and the corresponding components of α_{air} were found: the convective and radiative parts of heat transfer [6]. The value of T_s satisfying (8) was regarded as the true temperature. For an average air temperature of T_s = 20.75°C, over the 2.5-h period of the tests T_s turned out to be equal to 0.013°C under the conditions of the experiment.

The convective heat-transfer coefficient at the boundary of the air medium with the liquid film is determined using the formula for the free motion of air along the surface of the ice plate [6]:

$$\alpha_c = \mathrm{Nu}\,\lambda/H, \tag{9}$$

where

$$\mathrm{Nu} = A\,(\mathrm{Gr}\,\mathrm{Pr})^n. \tag{10}$$

For air, in accordance with the experimental conditions, $\mathrm{Gr} = 1/303\cdot 0{,}8^3\cdot 9{,}81\cdot(20{,}75 - 0{,}013)/(15.13\cdot 10^{-6})^2 = 1501.63\cdot 10^6$, $\mathrm{Pr} = 0.703$. The product of the indicated quantities GrPr = $1501.63\cdot 10^6\cdot 0.703 = 1.06\cdot 10^9 > 10^9$, and therefore A = 0.133 and n = 0.33, while α_k = 4.294 W/m²·°C.

The radiative heat-transfer coefficient between air and the surface of the water film running down the plate is found from the formula

$$\alpha_r = \varepsilon C_0\,\frac{\left(\frac{T_a}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4}{T_a - T_s}. \tag{11}$$

Here the values of the temperatures T_a and T_s are expressed in °K, the quantity α_r = 4.912 W/m²·K.

The total heat-transfer coefficient

$$\alpha_{\mathrm{air}} = \alpha_c + \alpha_r \tag{12}$$

is numerically equal to α_{air} = 4.294 + 4.912 = 9.206 W/(m²·°C).

The average heat-transfer coefficient of the liquid film α according to the formula (7) α = 14798.34 W(m²·°C). The values of the heat fluxes are as follows from air to the water film flowing down the plate: q_{air} = 9.206 (20.75-0.013) = 190.90 W/m², from the water film to thin ice: q = 14798.34(0.013-0) = 192.38 W/m², from the water film to the ice. The indicated quantities differ by 0.8%. This indicates that the value of T_s is determined with adequate reliability.

Using the temperature T_s determined from the formula (6), we find the local values of the heat-transfer coefficients at the boundary of the liquid film and the ice surface in the sections x_1 = 0.1 and x_2 = 0.7 m: α_{x_1} = 18665, α_{x_2} = 11475 W/(m²·°C).

A calculation of the thawing of the ice in these sections according to the dependences [7]

$$\Delta\xi_x = \frac{\alpha_x (T_s - T_m) t}{\gamma_i L} \tag{13}$$

gave the following results for t = 2.5 h: $\Delta\xi_{x_1}=0.0071$; $\Delta\xi_{x_2}=0.0044$ m. The experimental data for these sections and the corresponding times are $\Delta\xi_{x_1}=0.0086$; $\Delta\xi_{x_2}=0.0052$ m. The experimental and computed values differ by approximately 20%.

Figure 2 shows the results of the tests and calculations according to the proposed dependences with identical conditions of thawing of the ice plate. The results obtained are qualitatively very similar, and the quantitative disagreements are explained by the fact that constant average values of the temperatures T_a and T_s were used along the entire height of the plate. In reality they are not constants.

The proposed procedure can be used in practical calculations of thawing and destruction of shore slopes, embankments, fills, and excavations in permafrost regions. In addition, if the surface is not strictly vertical but is inclined at an angle φ to the horizontal, then the heat transfer coefficient must be calculated from the dependence [5]:

$$\alpha_\varphi = \alpha_{\text{vert}} \sqrt[4]{\sin\varphi}. \tag{14}$$

NOTATION

q, heat flux, W/m²; α, heat-transfer coefficient, W/(m²·°C), T_a and T_s, temperature of the air and of the water film at the boundary with the air, °C; T_m, melting temperature of the ice, 0°C; λ, coefficient of thermal conductivity of the water film formed (0.56 W/(m²·°C), and of the air ($25.96\cdot10^{-3}$ W/(m²·°C); δ, thickness of the water film formed, m; μ, coefficient of dynamic viscosity of the water film formed $17.88\cdot10^{-6}$ Pa·sec; γ_i, density of the ice, 917 kg/m³; γ, density of the water film formed, 1000 kg/m³; L, specific heat of fusion of the ice, $333.27\cdot10^3$ J/kg; H, height of the ice plate, m; Nu = αH/λ, Nusselt's number; Gr = $\beta g H^3 (T_a - T_s)/\nu^2$, Grashof number; Pr = ν/a, Prandtl's number; β, coefficient of volume expansion of air, 1/303 1/°C; g, acceleration of gravity, m/sec²; ν, coefficient of kinematic viscosity of air, $15.13\cdot10^{-6}$ m²/sec; a, thermal diffusivity of air, $21.52\cdot10^{-6}$ m²/sec; ε, emissivity of water, 0.95; C_0, radiation coefficient of an absolute blackbody, 5.67 W/m²·K⁴; Δξ, thickness of the thawing of the ice, m; and t, time, sec.

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